ADVANCED NODE FORMULATIONS IN TLM – THE ADAPTABLE SYMMETRICAL CONDENSED NODE (ASCN)

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Abstract – The paper describes the development of the adaptable symmetrical condensed TLM node (ASCN). The node exploits the opposing contributions to dispersion of stubs and link lines to minimize numerical errors. It is shown that this approach is effective and can be used to optimize nodal properties according to problem requirements.

1 INTRODUCTION

The process of discretization in space, inherent to all numerical solution schemes, introduces propagation errors in the transmission-line modelling (TLM) method. Additional propagation errors are introduced when modelling non-uniform materials and/or using non-cubic nodes (graded mesh). These errors decrease as space resolution increases, but there are practical limitations, brought about by run-time and storage requirements, to the extent that accuracy can be improved in this way. Hence, other approaches should be employed to develop more accurate schemes.

Local deviation in material properties and changes of cell aspect ratio are described in the TLM symmetrical condensed node (SCN) by adding stubs and/or modifying the link line impedances [1]–[4]. From the unified formulation of the TLM parameters [5] it follows that an infinite set of SCN-based schemes can be developed. All these schemes can be unified through the formulation of a general symmetrical condensed node (GSCN) [6]. Applying additional constraints to the GSCN, the stub-loaded SCN [1], hybrid SCN (HSCN) [2], symmetrical super-condensed node (SSCN) [3] and matched SCN (MSCN) [4], as well as other new, hitherto unexplored, schemes, can be derived.

In this paper, we exploit the opposing contributions to dispersion of stubs and link lines observed in the available TLM nodes [7]–[9] to develop a class of novel nodes, referred to as the adaptable SCN (ASCN). It is shown that they possess advantageous dispersion properties and substantially improved numerical accuracy compared to previous nodes.

2 THEORETICAL DEVELOPMENT

We illustrate here the development of the ASCN for a uniform mesh of node spacing Δl, where two possible values of link line impedances, Z_n and Z_p, and open- and short-circuit stubs of admittance Y_o and impedance Z_s, respectively, are allowed. The TLM constitutive relations for the GSCN take the form:

$$Y_n + Y_p + \frac{Y_o}{2} = \varepsilon\frac{\Delta l}{\Delta t} \quad (1)$$

$$Z_n + Z_p + \frac{Z_s}{2} = \mu\frac{\Delta l}{\Delta t} \quad (2)$$

where $Y_n = 1/Z_n$ and $Y_p = 1/Z_p$. If the link lines model a proportion of the medium parameters denoted by $(w_\varepsilon\varepsilon, w_\mu\mu)$, where w_ε and w_μ are arbitrary dimensionless weights, it follows that:

$$Y_n + Y_p = w_\varepsilon\varepsilon\frac{\Delta l}{\Delta t} \quad (3)$$

$$Z_n + Z_p = w_\mu\mu\frac{\Delta l}{\Delta t} \quad (4)$$

Combining (3) with (4), the link line impedances are obtained as:

$$Z_n = Z_1 A \qquad Z_p = Z_1/A \quad (5)$$

where

$$Z_1 = Z_0\sqrt{\frac{w_\mu\mu_r}{w_\varepsilon\varepsilon_r}} \quad (6)$$

$$A = \sqrt{w_\varepsilon w_\mu\mu_r\varepsilon_r} + \sqrt{w_\varepsilon w_\mu\mu_r\varepsilon_r - 1} \quad (7)$$

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Inserting (3)–(4) into (1)–(2), the stub parameters are found as:

$$Y_o = 4Y\sqrt{\varepsilon_r\mu_r}(1-w_\varepsilon) \quad (8)$$
$$Z_s = 4Z\sqrt{\varepsilon_r\mu_r}(1-w_\mu) \quad (9)$$

where $Z = 1/Y = \sqrt{\mu/\varepsilon}$.

The parameters of the GSCN for a uniform mesh are therefore defined by eqns. (5)–(9) in terms of the arbitrary weights w_ε and w_μ. For example, the stub-loaded SCN is defined by (5)–(9) if $w_\varepsilon = 1/\varepsilon_r$ and $w_\mu = 1/\mu_r$. Similarly, selecting $w_\varepsilon = 1/(\varepsilon_r\mu_r)$ and $w_\mu = 1$, the HSCN is formulated. In case of the SSCN, the requirements are $w_\varepsilon = w_\mu = 1$, while for the MSCN we need $w_\varepsilon = w_\mu = 1/\sqrt{\varepsilon_r\mu_r}$.

The dispersion behaviour of the available condensed nodes can be interpreted through w_ε and w_μ. For example, dispersion in the stub-loaded SCN is dependent on the ratio ε_r/μ_r for $\varepsilon_r\mu_r = const$ [8], which is a direct result of w_ε and w_μ not being functions of the product $\varepsilon_r\mu_r$. Similarly, two physical solutions to the dispersion relation, corresponding to different field polarizations, are experienced in the stub-loaded SCN and the HSCN [9], which is the consequence of selecting $w_\varepsilon \neq w_\mu$. As these observations are at variance with Maxwell's equations, we avoid them in the development of the ASCN by choosing $w_\varepsilon = w_\mu = w$, where w is a function of $\varepsilon_r\mu_r$.

Using the approach described in [8], the dispersion relation of the GSCN with $w_\varepsilon = w_\mu$ can be derived in an implicit polynomial form, allowing the study of dispersion in the GSCN as function of w. The SSCN and the MSCN can be considered as special cases of such a general SCN with $w_\varepsilon = w_\mu = w$. While in the stub-less SSCN all excess material parameters are modelled through the variation of link line impedances, in the MSCN it is done exclusively through the stubs. From the dispersion analysis it can be seen that these two approaches in modelling the variation in material parameters have opposing effects on propagation errors [4]. This is illustrated in Fig. 1 for three principal propagation directions. Since the SSCN is defined by $w = 1$ and the MSCN is defined by $w = 1/\sqrt{\varepsilon_r\mu_r}$ it is clear that by choosing $1/\sqrt{\varepsilon_r\mu_r} < w < 1$ a whole class of adaptable nodes may be obtained with propagation errors constrained between the errors for the SSCN and the MSCN.

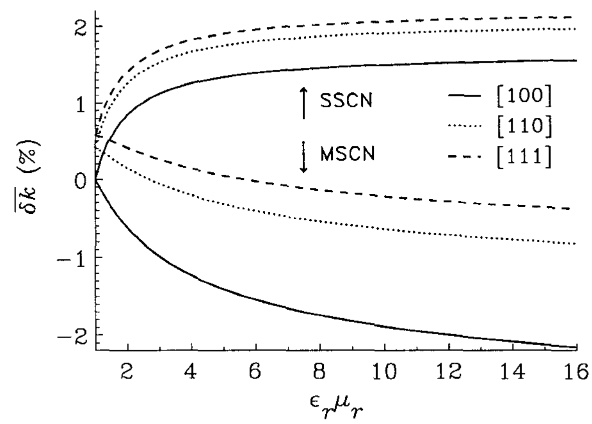

Figure 1: Percentage propagation error in the MSCN and the SSCN for $\Delta l/\lambda = 0.1$

Naturally, in order to compensate errors occuring in the MSCN and SSCN, the weighting function w of the ASCN should be chosen as a suitable mean function limited by $1/\sqrt{\varepsilon_r\mu_r}$ and unity. For example, the arithmetic and geometric means of these limits are:

$$w_a = \frac{\sqrt{\varepsilon_r\mu_r}+1}{2\sqrt{\varepsilon_r\mu_r}} \qquad \text{and} \qquad w_g = \frac{1}{\sqrt[4]{\varepsilon_r\mu_r}} \quad (10)$$

respectively. The weighting function w can be also selected according to problem requirements. For example, after a detailed analysis of the dispersion relation of the GSCN (not included here), a suitable function which minimizes errors for axial propagation and produces unilateral dispersion is found as:

$$w_u = \frac{1+2\varepsilon_r\mu_r}{3\varepsilon_r\mu_r} \quad (11)$$

Using the three functional forms of w described above, the propagation errors of the ASCN for three principal propagation directions are plotted in Fig. 2.

A comparison between the propagation errors of the ASCN, plotted in Fig. 2, and those of the SSCN and the MSCN plotted in Fig. 1, clearly demonstrates that the error in the ASCN, for any of the weighting functions used, is indeed contained between the errors in the MSCN and the SSCN. A direct result of this is that the error range in the ASCN with $w = w_a$ and particularly with $w = w_u$ is significantly smaller than in the SSCN and MSCN.

Fig. 3 shows the percentage propagation error (relative deviation in the propagation vector for a benchmark discretization of ten nodes per wavelength [8])

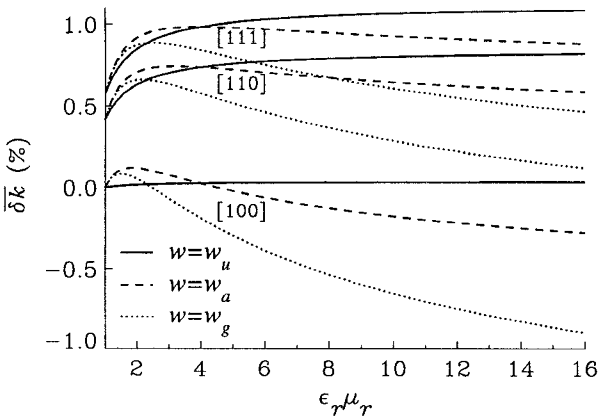

Figure 2: Percentage propagation error in the ASCN for $\Delta l/\lambda = 0.1$ using w_a, w_g and w_u

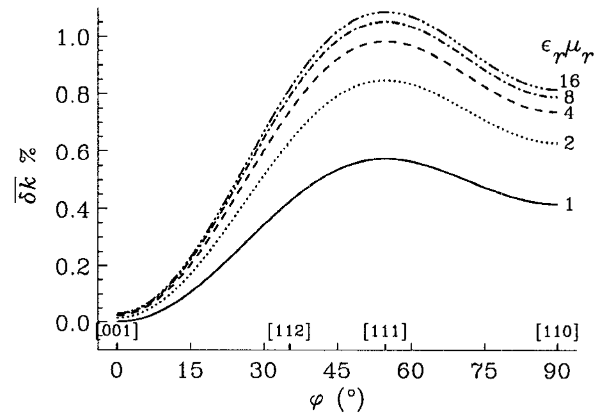

Figure 3: Percentage propagation error in the ASCN ($w = w_u$) for propagation along diagonal plane $x = y$ ($\Delta l/\lambda = 0.1$)

for the ASCN with $w = w_u$ for different values of $\varepsilon_r\mu_r$ and for propagation along a diagonal plane $x = y$. The error is plotted versus the angle φ formed by the propagation vector and the z-axis. In this case, propagation along directions $[mmn]$, such as [001], [110] and [111], can be studied. It can be seen from Fig. 3 that errors in the ASCN with $w = w_u$ extend only in the positive direction (unilateral dispersion) and increase with an increase in $\varepsilon_r\mu_r$.

Fig. 4 compares the propagation errors for the ASCN ($w = w_u$) with those obtained using the MSCN and the SSCN, for propagation along the diagonal plane $x = y$. It is clear that a substantial reduction in propagation errors is obtained with the ASCN, allowing for a more accurate modelling of non-uniform problems in TLM than was allowed by the previous nodes.

Using the definition of the GSCN [6], the development of the adaptable nodes presented here for the uniform mesh, can be readily generalized for a mesh with cuboid (graded) nodes. The optimization of the nodal properties can be achieved, in this case, using weighting functions in terms of $\Delta t/\Delta t_{\max}$, where Δt is the actual time step of the mesh, while $\Delta t_{\max}$ is the maximum permissible time-step on which the particular node can operate [3]. In the case of a cubic node mesh, when the time step Δt is chosen according to the background properties, it follows that

$$\frac{\Delta t}{\Delta t_{\max}} = \frac{1}{\sqrt{\varepsilon_r\mu_r}} \tag{12}$$

It is clear therefore that the weighting functions de-

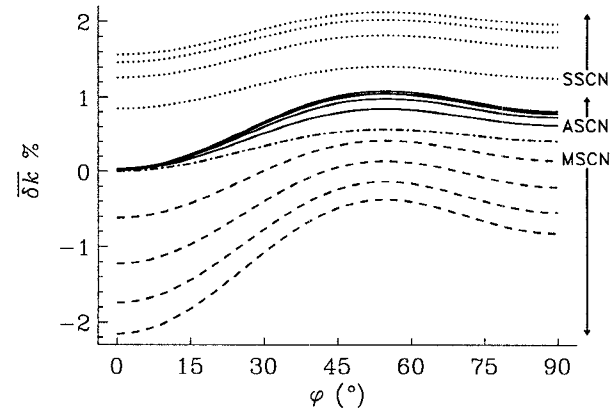

Figure 4: Comparison of propagation errors in the ASCN (solid lines), the SSCN (dotted lines) and the MSCN (broken lines) for $\Delta l/\lambda = 0.1$ and $\varepsilon_r\mu_r \in \{2, 4, 8, 16\}$ (direction of the increase in $\varepsilon_r\mu_r$ is denoted by arrows)

scribed by eqns. (10)–(11) can be rewritten in terms of $\Delta t/\Delta t_{\max}$, according to eqn. (12), and used in the definition of the graded ASCN.

3 NUMERICAL EXAMPLES

In order to validate numerical properties of the adaptable nodes, we modelled a partially filled canonical waveguide ($a = 2.286$cm, $b = 1.016$cm, $h = b/3$, $\varepsilon_r = 2.56$), depicted in Fig. 5, using a uniform TLM mesh with node spacing $\Delta l = b/12$. We performed TLM simulations using adaptable nodes with weighting functions $w = w_a$, $w = w_u$, $w = 1$ (the SSCN), $w = 1/\sqrt{\varepsilon_r\mu_r}$ (the MSCN), as well as using the HSCN (which in this case equals the traditional

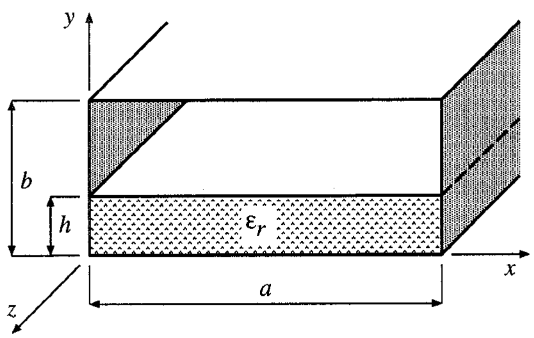

Figure 5: Partially loaded rectangular waveguide

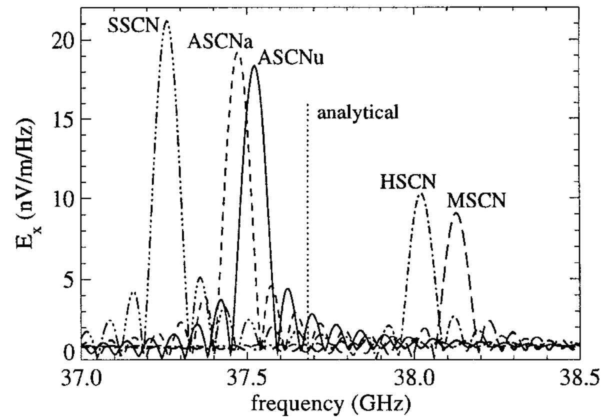

Figure 6: Frequency response of the waveguide around cutoff frequency of TE^y_{03} mode

stub-loaded SCN). A summary of analytical and simulated cutoff frequencies of two lower frequency hybrid modes are presented in Table 1.

	TE^y_{01}		TM^y_{11}	
	f[GHz]	δf [%]	f[GHz]	δf [%]
Analyt.	12.612		13.669	
SSCN	12.597	-0.119	13.652	-0.124
$ASCN^{w_a}$	12.604	-0.063	13.665	-0.029
$ASCN^{w_u}$	12.606	-0.048	13.667	-0.015
HSCN	12.626	+0.111	13.683	+0.102
MSCN	12.631	+0.151	13.693	+0.176

Table 1: Cutoff frequencies

It can be seen from Table 1 that the simulated frequencies for the ASCN are always between the results obtained with the SSCN and the MSCN and that they are the most accurate. Results obtained by modelling a higher cutoff frequency, e.g. for TE^y_{03} mode, where only around five nodes per wavelength are used and hence numerical dispersion is increased, are highlighted in Fig. 6. The improved accuracy of the ASCN, both with $w = w_a$ (ASCNa) and with $w = w_u$ (ASCNu) can be observed again. Note that the relative frequency error (calculated in the examples of this section) and the propagation error (plotted in Figs. 1–4), have opposite signs [8].

4 CONCLUSION

Using the framework of the general symmetrical condensed node (GSCN), we have developed a class of new adaptable nodes (ASCN), whose numerical properties can be customized by an arbitrary weighting function. Some of the possible weighting functions which optimize dispersion properties and minimize propagation errors were investigated. The improved accuracy of these schemes was illustrated by the example of modelling an inhomogeneous waveguide.

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